CBCS SCHEME

USN | | | | | | | | | | 17MAT41

# Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
## Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***





## Module-1

1 a. Find the Taylor series method, the value of y at x = 0.1 to five decimal places from $\frac{dy}{dx} = x^2y - 1$, y(0) = 1. Consider upto 4th degree terms. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y(0) = 1 and hence find y(0.1) by taking one step using Runge-Kutta method of fourth order. **(07 Marks)**

c. Given $\frac{dy}{dx} = \frac{x+y}{2}$, given that y(0) = 2, y(0.5) = 2.636, y(1) = 3.595, y(1.5) = 4.968 then find the value of y at x = 2 using Milne's method. **(07 Marks)**

**OR**

2 a. Using modified Euler's method, solve $\frac{dy}{dx} = x + |\sqrt{y}|$ with y(0) = 1 and hence find y(0.2) with h = 0.2. Modify the solution twice. **(06 Marks)**

b. Use fourth order Runge-Kutta method to find y(0.2), given $\frac{dy}{dx} = 3x + y$, y(0) = 1. **(07 Marks)**

c. Find y at x = 0.4 given $\frac{dy}{dx} + y + xy^2 = 0$ at $y_0 = 1$, $y_1 = 0.9008$, $y_2 = 0.8066$, $y_3 = 0.722$ taking h = 0.1 using Adams-Bashforth method. **(07 Marks)**

## Module-2

3 a. Given $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$. Find y at x = 0.2. Correct to four decimal places, given y = 1 and y′ = 0 when x = 0 using Runge-Kutta method. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x) = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **(07 Marks)**

**OR**

4 a. Given $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$, y(0) = 1, y′(0) = 1, compute y(0.4) for the following data, using Milne's predictor-corrector method.
y(0.1) = 1.1103, y(0.2) = 1.2427, y(0.3) = 1.399
y′(0.1) = 1.2103, y′(0.2) = 1.4427, y′(0.3) = 1.699 **(06 Marks)**

b. Express $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial. **(07 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2 - 1)^n]$ **(07 Marks)**

## Module-3

5 a. State and prove Cauchy-Rieman equation in Cartesian form. **(06 Marks)**

b. Evaluate $\int_C \frac{e^{2z}}{(z+2)(z+4)(z+7)}.dz$ where C is the circle |z| = 3 using Cauchy's residue theorem. **(07 Marks)**

c. Discuss the transformation $W = e^Z$. **(07 Marks)**

**OR**

6 a. Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **(06 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Find bilinear transformation which maps Z = i, 1, –1 onto W = 1, 0, ∞ **(07 Marks)**

## Module-4

7 a. A random variable X has the following probability function for various values of x:

| X (= xi) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(x) | 0 | K | 2K | 2K | 3K | $K^2$ | $2K^2$ | $7K^2$+K |

Find: (i) The value of K (ii) $P(x < 6)$ (iii) $P(x \geq 6)$ **(06 Marks)**

b. Derive mean and variance of the binomial distribution. **(07 Marks)**

c. The joint probability distribution of two random variables X and Y as follows:

| X \ Y | –4 | 2 | 7 |
|---|---|---|---|
| 1 | 1/8 | 1/4 | 1/8 |
| 5 | 1/4 | 1/8 | 1/8 |

Determine: (i) Marginal distribution of X and Y (ii) Covariance of X and Y (iii) Correlation of X and Y **(07 Marks)**

**OR**

8 a. In a certain factory turing out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing: (i) no defective (ii) one defective (iii) two defective blades, in a consignment of 10,000 packets. **(06 Marks)**

b. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. Given $p(0 < z < 1.2263) = 0.39$ and $p(0 < z < 1.4757) = 0.43$. **(07 Marks)**

c. Given:

| X \ Y | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1/8 | 1/4 | 1/8 |
| 1 | 1/8 | 1/4 | 1/8 | 0 |

Find : (i) Marginal distribution of X and Y (ii) E[X], E[Y], E[XY] **(07 Marks)**

## Module-5

**9** a. Define the terms:
(i) Null hypothesis
(ii) Confidence interval
(iii) Type-I and Type-II errors **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5, 3, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f is 2.201) **(07 Marks)**

c. Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$. Find the fixed probability vector. **(07 Marks)**

**OR**

**10** a. A die thrown 9000 times and a thrown of 3 or 4 was observed 3240 times. Is it reasonable to think that the die is an unbiased one? **(06 Marks)**

b. Four coins are tossed 100 times and the following results were obtained:

| Number of Heads | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Frequency | 5 | 29 | 36 | 25 | 5 |

Fit a binomial distribution for the data and test the goodness of fit [ $\chi^2_{0.05} = 9.49$ for 4 d.f]. **(07 Marks)**

c. Every year, a man trades for his car for a new car. If he has Maruti, he trade it for a Tata. If he has a Tata, he trade it for a Honda. However, if he has a Honda, he is just as likely to trade it for a new Honda as to trade it for a Maruti or a Tata. In 2016, he bought his first car which was a Honda. Find the probability that he has (i) 2018 Tata (ii) 2018 Honda (iii) 2018 Maruti. **(07 Marks)**

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